TN294_RC_dfln_crack_ex_12
051208

DEFLECTION OF CONCRETE MEMBERS USING NUMERICAL INTEGRATION

1 OVERVIEW

By way of a detailed numerical example, this Technical Note illustrates the application of equivalent moment of inertia "Ie" and numerical integration in calculating the post-cracking deflection of concrete members.

1.1 Background

Knowing the values listed below along the length of a span, the equivalent moment of inertia (Ie) at each point can be calculated;

$$I_e = (M_{cr} / M_a)^3 I_g + [1-(M_{cr} / M_a)^3] I_{cr} \leq I_g \qquad (1)$$

Where,

Icr	=	Moment of inertia of cracked section;
Ie	=	Effective moment of inertia;
Ma	=	Maximum moment in member at stage deflection is computed; and,
Mcr	=	Cracking moment.

1.2 Deflection Computation

Consider the two-span beam shown in **Fig 1-1**. The particulars of the beam are:

f'_c = 28 MPa
E_c = 24870 MPa
Concrete; normal weight = 2400 kg/m^3
Creep/shrinkage factor = 2.0
Steel yield strength = 460 MPa
Distance from tension fiber to centroid of tension steel = 71 mm
Distance from compression fiber to centroid of compression rebar = 71 mm
Cover = 60 mm top and bottom
Loading: applied dead load of 10 kN/m; concentrated dead load,
P = 25 kN at center of first span; live loading = 12 kN/m of beam.

The computed deflections by a software that accounts for cracking of sections (ADAPT-RC) are given in **Fig 1-2** graphically. The graphs show the deflection values at $1/20^{th}$ points along each span. The deflection values are also listed in **Table 1-1** for the first span.

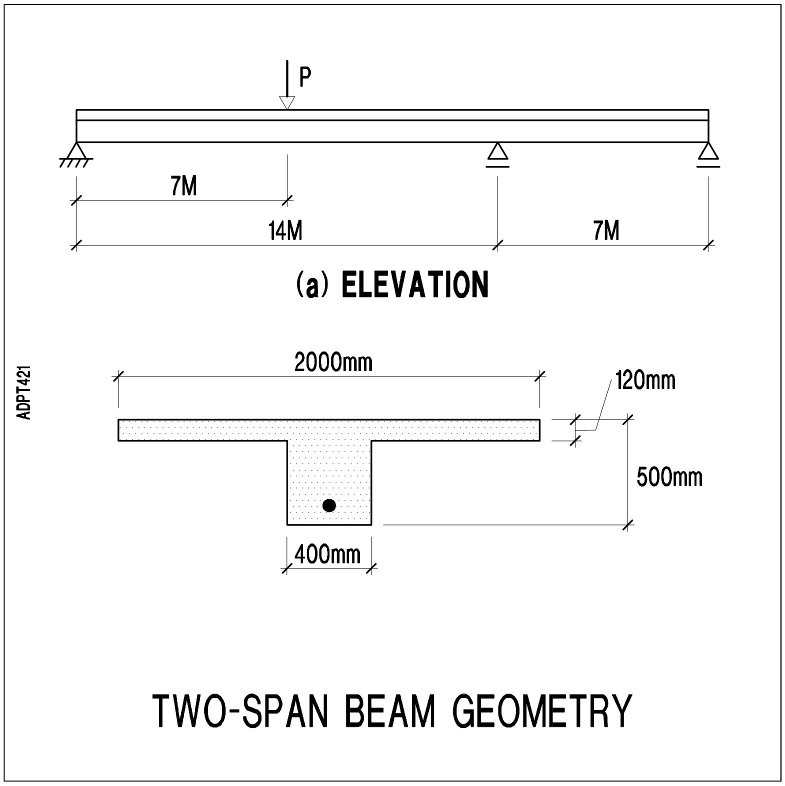

FIGURE 1-1

The numerical integration for deflection calculation is carried out span by span and is based on slope-deflection procedure as described next.

Refer to **Fig 1-3**. The distance, t, offset to the tangent at A, from point B, is given by the moment of the bending moment diagram between points A and B taken about point B divided by the respective E*I.

$$t = (\text{moment of the bending moment diagram about B})/(E_c * I_e)$$

$$\theta_a = t / AB$$

$$(w + t') = \theta_a * a$$

where, a, is the distance from support, A, to the location where deflection w is to be calculated.

The offset, t', at location of where deflection is to be calculated is given by:

$$t' = (\text{moment of the bending moment diagram between A, and D, about, D})/(E_c * I_e)$$

TABLE 1-1 ADAPT-RC COMPUTED DEFLECTIONS OF SPAN 1

X/L	X (mm)	Deflections, δ DL + LL (mm)
0.00	0	0.00
0.05	700	13.44
0.10	1400	26.50
0.15	2100	38.71
0.20	2800	49.86
0.25	3500	59.11
0.30	4200	66.75
0.35	4900	72.38
0.40	5600	76.00
0.45	6300	77.20
0.50	7000	76.40
0.55	7700	73.18
0.60	8400	68.36
0.65	9100	61.52
0.70	9800	53.48
0.75	10500	44.23
0.80	11200	34.69
0.85	11900	24.91
0.90	12600	15.41
0.95	13300	6.79
1.00	14000	0.00

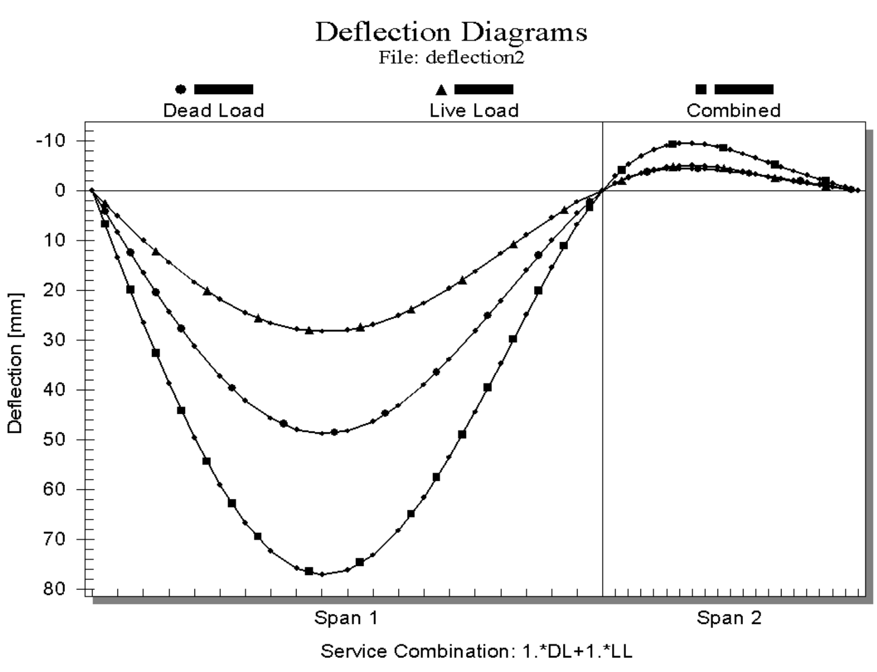

FIGURE 1-2

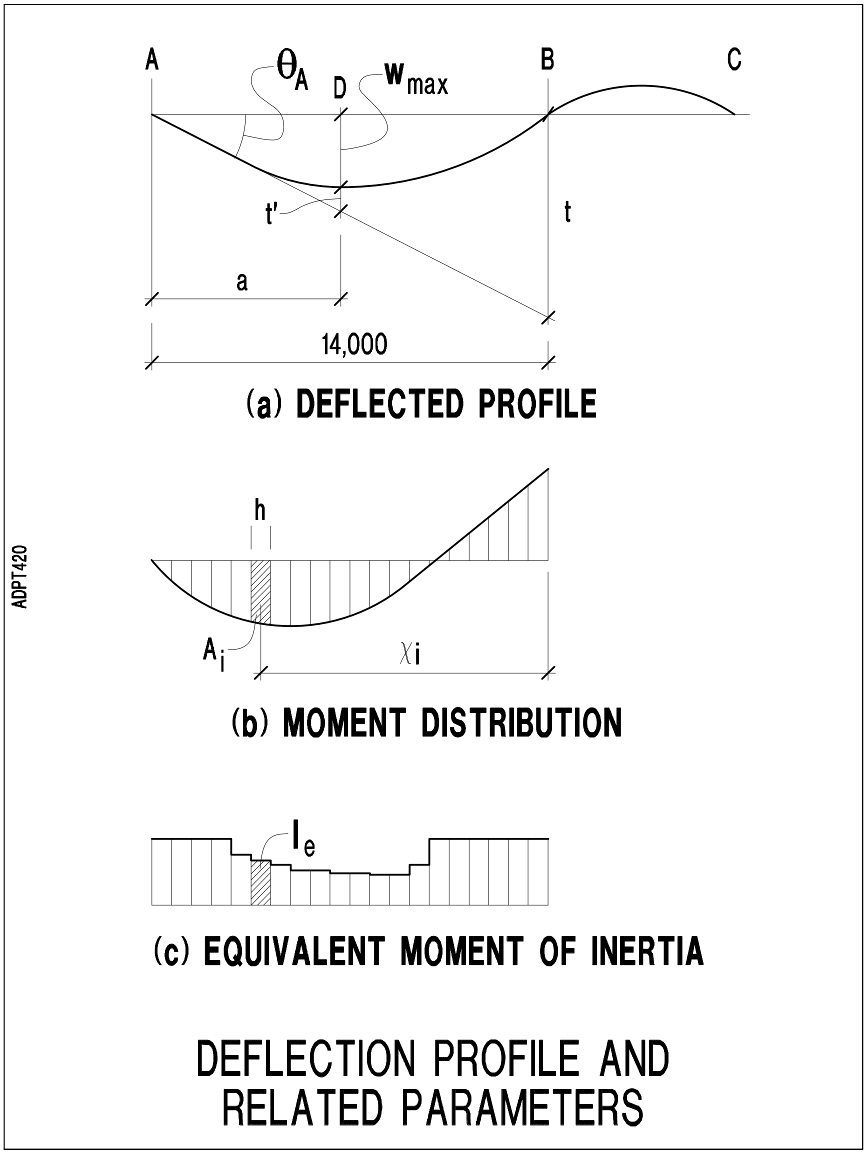

FIGURE 1-3

Hence

$w = \theta_a * a - t'$

Using the procedure described, "w" along the length of each span is calculated.

Dead and Live Load Deflection

For dead and live load deflection, the applied moment and the associated equivalent moment of inertia are due to the simultaneous application of dead and live loading. The values of the parameters involved are given in **Tables 1-2** and **1-3**.

$t = \Sigma A_i * X_i / (E_c * I_g) = 270.51$ mm

θ_a = 270.51 mm/14,000 mm = 1.93E-02

For a point 6300 mm from the left support, the values are:

$(w_{max} + t')$ = 1.93E-02*6,300 mm = 121.73 mm

$t' = \Sigma A'_i * X'_i / (E_c * I_g)$ = 45.41 mm

w due to dead and live loading

= 121.73 mm - 45.41 mm = 76.32 mm

This agrees with the deflection value reported by the program in Table 1-1

TABLE 1-2 MOMENT-AREA INTERIM COMPUTATIONS

X	Moment**	I_e**	A_i	C_i	X_i	X_i'	A_iX_i/E_cI_e	A_iX_i'/E_cI_e
(mm)	(N.mm)	(mm4)	(N.mm2)	(mm)	(mm)	(mm)	(mm)	(mm)
0	6165	7933400000	n/a	n/a	n/a	n/a	n/a	n/a
700	124500000	5732064000	43577157750	233.34	13533.34	5833.34	4.137	1.783
1400	233700000	5176954000	1.2537E+11	314.43	12914.43	5214.43	12.575	5.078
2100	327500000	5114004000	1.9642E+11	330.50	12230.50	4530.50	18.888	6.997
2800	406100000	5096920000	2.5676E+11	337.50	11537.50	3837.50	23.370	7.773
3500	469400000	5090278000	3.06425E+11	341.56	10841.56	3141.56	26.242	7.604
4200	517400000	5087187000	3.4538E+11	344.33	10144.33	2444.33	27.693	6.673
4900	550100000	5085657000	3.73625E+11	346.43	9446.43	1746.43	27.905	5.159
5600	567400000	5084984000	3.91125E+11	348.19	8748.19	1048.19	27.056	3.242
6300	569500000	5084908000	3.97915E+11	349.78	8049.78	349.78	25.329	1.101
7000	556300000	5157166000	3.9403E+11	351.37	7351.37	-348.63	22.585	n/a
7700	510200000	5159284000	3.73275E+11	355.04	6655.04	-1044.96	19.360	n/a
8400	448900000	5163620000	3.35685E+11	357.46	5957.46	-1742.54	15.573	n/a
9100	372200000	5173880000	2.87385E+11	360.90	5260.90	-2439.10	11.750	n/a
9800	280300000	5205883000	2.28375E+11	366.43	4566.43	-3133.57	8.055	n/a
10500	173000000	5387626000	1.58655E+11	377.62	3877.62	-3822.38	4.591	n/a
11200	-554000	7933400000	60356100000	467.42	3267.42	-4432.58	1.000	n/a
11900	-108200000	7933400000	-38063900000	234.52	2334.52	-5365.48	-0.450	n/a
12600	-262500000	5134348000	-1.29745E+11	301.44	1701.44	-5998.56	-1.729	n/a
13300	-432200000	4390003000	-2.43145E+11	321.50	1021.50	-6678.50	-2.275	n/a
14000	-617200000	4248884000	-3.6729E+11	329.43	329.43	-7370.57	-1.145	n/a
						S =	270.509	45.409

E_c = Modulus elasticity of concrete = 24870.00 N/mm^2.
A_i = Moment area each 1/20th subdivision (assumed as a trapezoid) (see **Fig 1-3**)
Ci = Centroid of each 1/20th subdivision (assumed as a trapezoid)
X_i = Moment arm of each 1/20th subdivision about point B (see **Fig 1-3**)
X_i' = Moment arm of each 1/20th subdivision about point D (see **Fig 1-3**)

TABLE 1-3 ADAPT-RC MOMENTS AND MOMENT OF INERTIAS DUE TO DEAD AND LIVE LOAD

ADAPT STRUCTURAL CONCRETE SOFTWARE SYSTEM

Applied moment (Ma), Cracked moment (Mcr), Gross Moment of Inertia (Ig)
Cracked I (Icr) and Effective I (Ie) for span 1

S	pts	Ma Nmm	Mcr Nmm	Ig mm4	Icr mm4	Ie mm4
1	0	.6165000E+04	.7617415E+08	.79334000E+10	.50780750E+10	.79334000E+10
1	1	.1245000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.57320640E+10
1	2	.2337000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.51769540E+10
1	3	.3275000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.51140040E+10
1	4	.4061000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.50969200E+10
1	5	.4694000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.50902780E+10
1	6	.5174000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.50871870E+10
1	7	.5501000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.50856570E+10
1	8	.5674000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.50849840E+10
1	9	.5695000E+09	.7617415E+08	.79334000E+10	.50780750E+10	.50849080E+10
1	10	.5563000E+09	.7617415E+08	.79334000E+10	.51500200E+10	.51571660E+10
1	11	.5102000E+09	.7617415E+08	.79334000E+10	.51500200E+10	.51592840E+10
1	12	.4489000E+09	.7617415E+08	.79334000E+10	.51500200E+10	.51636200E+10
1	13	.3722000E+09	.7617415E+08	.79334000E+10	.51500200E+10	.51738800E+10
1	14	.2803000E+09	.7617415E+08	.79334000E+10	.51500200E+10	.52058830E+10
1	15	.1730000E+09	.7617415E+08	.79334000E+10	.51500200E+10	.53876260E+10
1	16	-.5540000E+06	-.1665114E+09	.79334000E+10	.44118250E+10	.79334000E+10
1	17	-.1082000E+09	-.1665114E+09	.79334000E+10	.44118250E+10	.79334000E+10
1	18	-.2625000E+09	-.1665114E+09	.79334000E+10	.41750850E+10	.51343480E+10
1	19	-.4322000E+09	-.1665114E+09	.79334000E+10	.41750850E+10	.43900030E+10
1	20	-.6172000E+09	-.1665114E+09	.79334000E+10	.41750850E+10	.42488840E+10

1.3 Computation of Equivalent Moment of Inertia, I_e

Refer to ADAPT TN293 for the calculation of Ie. This Technical Note uses the formulation given in TN293 and illustrates the calculation of Ie for a selected point along the first span of the above example.

In addition to the geometry of the section, the location and amount of both the tension and the compression reinforcement are necessary to compute I_{cr}. Herein, I_{cr} at the section next to the second support is hand calculated. This section contains both negative and positive reinforcement.

Since tension occurs at the top fiber of the section, indicated by the negative moment in **Table 1-3**, the following are the primary equations used in determining the cracking moment of inertia, I_{cr}.

$$I_{cr} = bk^3d^3/3 + nA_s(d - kd)^2 + A_s'(n-1)(kd-d')^2$$

Where,

$$c = kd = \{[2dB(1 + rd'/d) + (1 + r)^2]^{1/2} - (1 + r)\}/B$$

and,

$$B = b/(nA_s)$$

$$r = (n - 1)\,A_s'/(nA_s)$$

For the current problem,

A_s = 6521 mm^2
A_s' = 3156mm^2
b = 400 mm
d = 429 mm
d' = 71 mm
E_c = 24,870 N/mm^2
E_s = E_s' = 200,000 N/mm^2
n = E_s/E_c = 8.04

Solving for B and r, to obtain c,

B = 400/(8.04*6521)
= 7.629E-03 /mm

r = (8.04 - 1)*3156/8.04*6521
= 0.424

c = kd = {[2*429*7.631E-03*(1 + (0..424*71/429)) + $(1 + 0.424)^2]^{1/2}$ - (1 + 0.424)}/ 7.629E-03
= 207.28 mm

(ADAPT-RC 207.33 mm)

And finally, solving for I_{cr},

I_{cr} = 400*$(207.28)^3$/3 + 8.04*6521*$(429 - 207.28)^2$ + 3156(8.04 - 1)*$(207.28 - 71)^2$
= 0.4177E+10mm^4

(ADAPT-RC .4175E+10 mm^4, Table 1-3)

$$I_e = (M_{cr} / M_a)^3 I_g + [1-(M_{cr} / M_a)^3]I_{cr} \leq I_g$$

where for the current problem,

M_{cr} = -.1665114E+09N.mm (Extracted from Table 1-3)

M_a = -.2625000E+09N.mm (Extracted from Table 1-3)

I_g = 0.7933400E+10 mm^4 (Extracted from Table 1-3)

After substitution,

I_e = (-0.1665114E+09 / -0.2625000E+09 $)^3$ *0.7933400E+10 + [1-(-0.1665114E+09 / -0.2625000E+09$)^3$]*0.4177E+10
= 0.5136E+10 mm^4 < I_g = 0.79334E+10 mm^4

(ADAPT-RC 0.51343480E+10 mm^4, Table 1-3)